

# Energy dependence of hadron polarization in $e^+e^- \rightarrow hX$ at high energies

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The longitudinal polarization of hyperon in  $e^+e^-$  annihilation at high energies depends on the longitudinal polarization of the quark produced at the  $e^+e^-$  annihilation vertex whereas the spin alignment of vector mesons is independent of it. They exhibit very different energy dependences. We use the longitudinal polarization of Lambda hyperon and the spin alignment of  $K^*$  as examples and present numerical results of energy dependences. We present the results at the leading twist with perturbative QCD evolutions of fragmentation functions at the leading order.

## I. INTRODUCTION

The spin dependence of fragmentation functions (FFs) has attracted much attention since it provides not only important information on hadronization mechanism but also an important place to study properties of quantum chromodynamics (QCD). High energy  $e^+e^-$  annihilation is the cleanest place to study FFs. Among different aspects of the spin dependence, vector polarizations of hyperons and tensor polarizations of vector mesons are two topics that attracted special attention because both of them can be measured by the angular distributions of the decay products. Hyperon polarizations can be determined by the angular distribution of the decay products of the spin self-analyzing parity violating weak decay. Different components of the tensor polarization of vector mesons can also be determined by the angular distribution of decay products of the strong decay into two spin zero hadrons. Measurements have been carried out e.g. many years ago at LEP for the longitudinal polarization of  $\Lambda$  hyperon [1, 2] and for spin alignments of vector mesons [3–5] in the inclusive production process  $e^+e^- \rightarrow hX$  and sizable effects have been observed. These data have attracted many phenomenological studies and different approaches have been proposed to describe them [6–24].

In the theoretical framework of the QCD parton model, hadron polarizations are expressed in terms of different FFs [25–31]. These FFs are defined via quark-quark and/or quark-gluon-quark correlators. The results of the complete decomposition of quark-quark correlator as well as those for that of quark-gluon-quark at twist-3 for spin-1 hadrons has been presented e.g. in [31–33]. A general framework for  $e^+e^- \rightarrow V\pi X$  has been constructed [31] and QCD parton model results for hadron polarizations in terms of FFs have been presented up to twist-3.

With these results, we can make predictions on the energy dependence of hadron polarization within the theoretical framework of QCD if we know the results at a given energy. In fact, from the results presented in [25–31], we see one distinct feature for hadron polarizations in  $e^+e^-$  annihilations at high energies, i.e., at the leading twist, polarizations of hadrons are divided into two categories. In one of them, the polarization of hadrons depends on the initial longitudinal polarization  $P_q$  of the quark (or anti-quark) produced at the  $e^+e^-$ -vertex and is parity violated. In the second category, the polarization is independent of  $P_q$  and is parity conserved. The most well-known example is the longitudinal polarization of hyperons such as  $\Lambda$ ,  $\Sigma$  and  $\Xi$ , while spin alignments of vector mesons such as  $\rho$  and  $K^*$  are representatives of the second category. The longitudinal polarization  $P_q$  is a result of weak interaction and is completely determined by the

electro-weak process at the parton level. It takes the maximum for  $e^+e^-$  annihilation at the Z-pole and changes very fast with energy. Hence, we expect that the polarization in the first category has strong energy dependence. The energy dependence for hadron polarizations in the second category comes mainly from the scale dependence of the corresponding FFs and/or higher twist contributions. We expect that they change quite slowly with energy compared with that in the first category. We should see very much different behaviors in energy dependence.

Clearly, the energy dependence provides not only a good place to study the spin dependence of FF but also a good place to study QCD evolution of the spin-dependent FF and higher twist contributions. In view that there are some data available from experiments at LEP [1–5] and that new measurements can be carried out in experiments at very much different energies such as BES III and BELLE [34] and possibly at the future facilities planned and/or discussed [35], it is very interesting to present some numerical results to guide experiment and test models.

In this paper, after a brief summary of hadron polarizations in terms of FFs in  $e^+e^- \rightarrow hX$ , we take the longitudinal polarization of  $\Lambda$  and the spin alignment of  $K^*$  as two representative examples for the two categories and calculate the energy dependence. We take them as examples because we have data from LEP for both of them. We make a simple working parameterization for the corresponding FFs by fitting the LEP data [1–5], and evolve them to other energies. We present the results numerically that can be used as a rough guide for future experiments.

The rest of the paper is organized as follows. After this introduction, we summarize the results of FFs defined via quark-quark correlator, those for hadron polarizations in terms of FFs in  $e^+e^- \rightarrow hX$  and QCD evolution equations for FFs in Sec. II. In sec. III, we present a working parameterization of the corresponding FFs, show the numerical results of QCD evolution at the leading order and present the energy dependence of the two representative examples. We make a short summary and an outlook in Sec. IV.

## II. HADRON POLARIZATIONS IN $e^+e^- \rightarrow hX$ IN TERMS OF FFs

High energy  $e^+e^- \rightarrow hX$  is the best place to study FFs in different connections. The results for hadron polarizations expressed in terms of FFs up to twist-3 in leading order in pQCD are given in different papers such as [25–31]. Here, we make a short summary of these results and present in particular the formulae that will be used in the numerical estimations.

### A. FFs defined via quark-quark correlator

The polarization of hadron produced in high energy reaction is described by the spin density matrix. For spin-1/2 hadrons, the polarization is described by a  $2 \times 2$  spin density matrix that is usually decomposed as  $\rho = (1 + \vec{S} \cdot \vec{\sigma})/2$ , where  $\vec{\sigma}$  is the Pauli matrix, and  $\vec{S}$  is the polarization vector which is represented by the helicity  $\lambda$  and the transverse polarization vector  $S_T^\mu$ , i.e.,

$$S^\mu = \lambda \frac{p^+}{M} \bar{n}^\mu + S_T^\mu - \lambda \frac{M}{2p^+} n^\mu, \quad (1)$$

where  $n$  and  $\bar{n}$  are the two unit vectors in light-cone coordinates. For spin-1 hadrons, the polarization is described by a  $3 \times 3$  density matrix, which, in the rest frame of the hadron, is usually decomposed as [36]

$$\rho = \frac{1}{3}(\mathbf{1} + \frac{3}{2}S^i \Sigma^i + 3T^{ij} \Sigma^{ij}), \quad (2)$$

where  $\Sigma^i$  is the spin operator of spin-1 particle, and  $\Sigma^{ij} = \frac{1}{2}(\Sigma^i \Sigma^j + \Sigma^j \Sigma^i) - \frac{2}{3}\mathbf{1}^{ij}$ . The spin polarization tensor  $T^{ij} = \text{Tr}(\rho \Sigma^{ij})$ , and is parameterized as

$$\mathbf{T} = \frac{1}{2} \begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}. \quad (3)$$

The tensor polarization part has five independent components that are given by a Lorentz scalar  $S_{LL}$ , a Lorentz vector  $S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0)$  and a Lorentz tensor  $S_{TT}^{\mu\nu}$  that has two nonzero independent components  $S_{TT}^{xx} = -S_{TT}^{yy}$  and  $S_{TT}^{xy} = S_{TT}^{yx}$ .

For the fragmentation of the quark (or anti-quark), the FFs are defined via the quark-quark and/or the quark-gluon-quark correlators. The quark-quark or quark-gluon-quark correlator can in general be expressed as a sum of a spin-independent part, a vector polarization dependent part and a tensor polarization dependent part. To describe the production of spin zero hadrons, we need only the spin-independent part. For spin-1/2 hadrons, the vector-polarization dependent part is involved, and for spin-1 hadrons, the tensor polarization dependent part is also needed. FFs are obtained by making Lorentz decompositions of the corresponding part in terms of 4-momenta and variables describing the polarization. Hence, formally, the spin independent part is exactly the same for hadrons with different spins, the vector polarization dependent part is also the same for spin-1/2 and spin-1 hadrons.

The results for the complete decomposition of quark-quark correlator are summarized e.g. in [31]. At the leading twist, there are totally 18 TMD FFs that are summarized in Table II of [31]. From the table, we see that 5 of these 18 leading twist TMD FFs describe fragmentation of unpolarized, 4 of them describe longitudinally polarized and 9 of them describe transversely polarized quark. For those describing unpolarized quark fragmentation, we have the well-known  $D_1(z, k_\perp)$  describing the number density of hadrons produced in the fragmentation and the other 4 describing the induced polarizations. Similarly, for FFs of the longitudinally and transversely polarized quark, we have the direct spin transfer  $G_{1L}$  and  $H_{1T}$  respectively and others describing the number density and/or “worm-gear effects”.

After integrating over the transverse momentum, we obtain the results in the one-dimensional case. In this case, we have only five FFs left at the leading twist, i.e., the number density  $D_1(z)$ , the induced  $D_{1LL}(z)$ , the direct spin transfers in the longitudinally polarized case  $G_{1L}(z)$ , and in the transversely polarized case  $H_{1T}(z)$  and  $H_{1LT}(z)$ .

We emphasize that one-dimensional FFs are needed to describe inclusive processes such as  $e^+e^- \rightarrow hX$  while three-dimensional FFs are needed for semi-inclusive processes such as  $e^+e^- \rightarrow h_1 h_2 X$ . They can be studied in the corresponding processes respectively. Also, to study those FFs for unpolarized, transversely polarized or longitudinally polarized quarks, one needs to create quarks in the corresponding polarization states and know the polarizations of them before the fragmentation.

### B. Quark polarization in $e^+e^- \rightarrow q\bar{q}$

It is well known that the quark or anti-quark from  $e^+e^- \rightarrow Z \rightarrow q\bar{q}$  is longitudinally polarized. The polarization is given by

$$P_q^{Zpole}(\theta) = -\frac{c_1^e c_3^q (1 + \cos^2 \theta) + 2c_3^e c_1^q \cos \theta}{c_1^e c_1^q (1 + \cos^2 \theta) + 2c_3^e c_3^q \cos \theta}, \quad (4)$$

where  $\theta$  is the angle between the incident electron and the produced quark,  $c_1^e = (c_V^e)^2 + (c_A^e)^2$ ,  $c_3^e = 2c_V^e c_A^e$ ,  $c_V^e$  and  $c_A^e$  are defined in the weak interaction current  $\bar{\psi} \gamma^\mu (c_V^e - c_A^e \gamma^5) \psi$  and the superscript denotes that they are for the electron, and similarly for different flavors of quarks.

Although the quark (anti-quark) is not transversely polarized, their transverse spin components are correlated. This is described by the transverse spin correlation function  $c_{nn}^q$  defined as

$$c_{nn}^q \equiv \frac{|\hat{m}_{n++}|^2 + |\hat{m}_{n--}|^2 - |\hat{m}_{n+-}|^2 - |\hat{m}_{n-+}|^2}{|\hat{m}_{n++}|^2 + |\hat{m}_{n--}|^2 + |\hat{m}_{n+-}|^2 + |\hat{m}_{n-+}|^2}, \quad (5)$$

where  $\hat{m}$  is the scattering amplitude, + or – denotes that the quark or anti-quark is in  $s_n = 1/2$  or  $-1/2$  state. If we take  $\vec{n}$  as the normal of the production plane, we obtain

$$c_{nn}^{q,Zpole}(\theta) = \frac{c_1^e c_2^q \sin^2 \theta}{c_1^e c_1^q (1 + \cos^2 \theta) + 2c_3^e c_3^q \cos \theta}, \quad (6)$$

where  $c_2^q = (c_V^q)^2 - (c_A^q)^2$ . Define  $y = l_2 \cdot p_q / q \cdot p_q \approx (1 + \cos \theta)/2$  ( $l_1$  and  $l_2$  are the 4-momenta of the incident  $e^-$  and  $e^+$ ,  $q = l_1 + l_2$  is that of the  $Z$ -boson, and  $p_q$  is that of the produced quark), we can express  $P_q$  and  $c_{nn}^q$  in terms of  $y$ , i.e.,

$$P_q^{Zpole}(y) = T_1^q(y)/T_0^q(y), \quad (7)$$

$$c_{nn}^{q,Zpole}(y) = c_1^e c_2^q C(y)/2T_0^q(y), \quad (8)$$

$$T_0^q(y) = c_1^e c_1^q A(y) - c_3^e c_3^q B(y), \quad (9)$$

$$T_1^q(y) = -c_1^e c_3^q A(y) + c_3^e c_1^q B(y). \quad (10)$$

Here we denote as usual  $A(y) = (1 - y)^2 + y^2 \approx (1 + \cos^2 \theta)/2$ ,  $B(y) = 1 - 2y \approx -\cos \theta$ , and  $C(y) = 4y(1 - y) \approx \sin^2 \theta$ .

Experimental studies are often carried out irrespective of  $\theta$  or  $y$ . The obtained results just correspond to the results integrated over  $\theta$  or  $y$ . For the polarization and correlation of quark given above, if we integrate over  $\theta$  or  $y$ , we obtain

$$\bar{P}_q^{Zpole} = -c_3^q/c_1^q, \quad (11)$$

$$\bar{c}_{nn}^{q,Zpole} = c_2^q/2c_1^q. \quad (12)$$

We see that, the quark is negatively polarized in the longitudinal direction. Also  $c_2 < 0$  since  $c_V^2$  is smaller than  $c_A^2$ , so we have a negative  $c_{nn}^q$  at the  $Z$ -pole.

In general, for  $e^+e^- \rightarrow q\bar{q}$ , we need to consider contributions from  $e^+e^- \rightarrow Z \rightarrow q\bar{q}$ , those from  $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$  and the interference terms. In this case, we have

$$P_q(y) = \Delta w_q(y)/w_q(y), \quad (13)$$

$$c_{nn}^q(y) = 2y(1 - y)(e_q^2 + \chi c_1^e c_2^q + \chi_{int}^q c_V^e c_V^q)/w_q(y). \quad (14)$$

Here  $e_q$  is the electric charge of  $q$ , and  $w_q(y)$  and  $\Delta w_q(y)$  are given by

$$w_q(y) = \chi T_0^q(y) + e_q^2 A(y) + \chi_{int}^q I_0^q(y), \quad (15)$$

$$\Delta w_q(y) = \chi T_1^q(y) + \chi_{int}^q I_1^q(y), \quad (16)$$

$$I_0^q(y) = c_V^e c_V^q A(y) - c_A^e c_A^q B(y), \quad (17)$$

$$I_1^q(y) = -c_V^e c_A^q A(y) + c_A^e c_V^q B(y), \quad (18)$$

$$\chi = s^2 / [(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^4 2\theta_W, \quad (19)$$

$$\chi_{int}^q = -2e_q s(s - M_Z^2) / [(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^2 2\theta_W, \quad (20)$$

where  $M_Z$  and  $\Gamma_Z$  are the mass and decay width of  $Z$ ,  $\theta_W$  is the Weinberg angle, and  $s = q^2 = Q^2$ .

After integrating over  $y$ , we obtain

$$\bar{P}_q = \Delta W_q / W_q, \quad (21)$$

$$\bar{c}_{nn}^q = (e_q^2 + \chi c_1^e c_2^q + \chi_{int}^q c_V^e c_V^q) / (3W_q), \quad (22)$$

where  $\Delta W_q$  and  $W_q$  are the results of  $\Delta w_q(y)$  and  $w_q(y)$  after integration over  $y$ , and they are given by

$$\Delta W_q = -\frac{2}{3}(\chi c_1^e c_3^q + \chi_{int}^q c_V^e c_A^q), \quad (23)$$

$$W_q = \frac{2}{3}(e_q^2 + \chi c_1^e c_1^q + \chi_{int}^q c_V^e c_V^q). \quad (24)$$

We see that both  $\bar{P}_q$  and  $\bar{c}_{nn}^q$  depend on the energy  $\sqrt{s}$ , and behave quite differently in the energy dependence. For comparison, we plot them in Fig. 1. We see clearly that, in the energy region  $\sqrt{s} \leq M_Z$ , as  $\sqrt{s}$  decreases, the electromagnetic interaction becomes dominate, the longitudinal polarization of quark  $\bar{P}_q$  goes to zero rapidly, but the correlation  $\bar{c}_{nn}^q$  goes from negative to positive and reaches the maximum 1/2 rapidly. For  $\sqrt{s} \geq M_Z$ , we have contributions from both weak and electromagnetic interactions, and they combine together to give rise to a negative  $P_q$  but positive  $c_{nn}^q$ . The correlation between the transverse spin components of the quark and anti-quark is strong and positive.

From these results we see in particular the following. In  $e^+e^-$ -annihilation at high energies, we have possibilities to study FFs of unpolarized, longitudinally polarized as well as transversely polarized quarks. First, we can study FFs of unpolarized or longitudinally polarized quarks by studying singly polarized reactions, i.e., by measuring only the polarization of one hadron in the final state. More precisely, we can study one-dimensional FFs of unpolarized or longitudinally

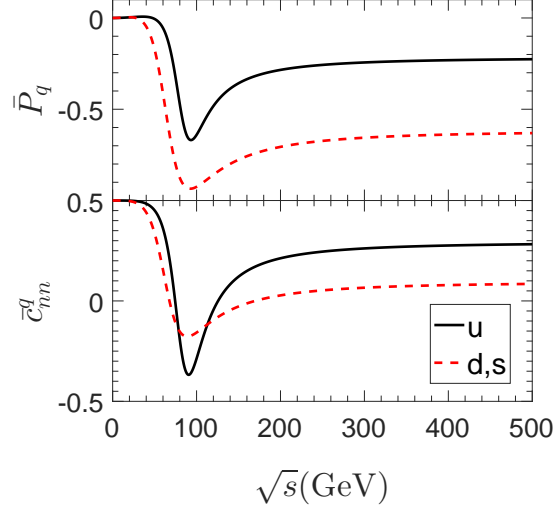


FIG. 1: Energy dependence of the longitudinal quark polarization and the transverse quark-anti-quark spin correlation produced in  $e^+e^-$  annihilation.

polarized quarks in  $e^+e^- \rightarrow hX$  by measuring the corresponding components of polarizations of  $h$  in the final state. By studying the semi-inclusive process  $e^+e^- \rightarrow h_1 h_2 X$  and measuring the polarization of  $h_1$ , we can study the corresponding three-dimensional FFs. Second, FFs of the transversely polarized quark can also be studied in  $e^+e^-$ -annihilation at high energies. But in this case, we need at least to measure polarizations or other spin dependent asymmetries of two hadrons in the final states since the nonzero quantity at the parton level is the transverse spin correlation between the initial quark and anti-quark but not the transverse polarization of the quark or anti-quark. In this paper, we start with the simplest case, i.e.  $e^+e^- \rightarrow hX$  where only one-dimensional FFs for the unpolarized or longitudinally polarized quark can be studied.

### C. Hadron polarizations at the Z-pole

Hadron polarizations in  $e^+e^-$ -annihilations at high energies are given e.g. in [29–31] in terms of FFs. For  $e^+e^- \rightarrow Z \rightarrow VX$  at the leading order in pQCD and up to twist-3, for the longitudinal components, we obtain

$$\langle \lambda \rangle(z, y) = \frac{2}{2S+1} \frac{\sum_q P_q(y) T_0^q(y) G_{1L}(z)}{\sum_q T_0^q(y) D_1(z)}, \quad (25)$$

$$\langle S_{LL} \rangle(z, y) = \frac{3}{2(2S+1)} \frac{\sum_q T_0^q(y) D_{1LL}(z)}{\sum_q T_0^q(y) D_1(z)}. \quad (26)$$



Here, for brevity and clarity, we omit the superscript  $q \rightarrow V$  in the fragmentation functions, e.g.,  $D_1(z) = D_1^{q \rightarrow V}(z)$ ; and  $S$  is the spin of hadron  $h$ . The factor  $(2S + 1)$  appears here because, in the conventions used in [31] in defining FFs via quark-quark correlator and/or quark-gluon-quark correlator,  $D_1(z)$  is the number density for the produced  $h$  averaging over rather than summing over the spin of  $h$ . We write this factor explicitly so that the corresponding expressions eventually take the same form for spin-1/2 as well as spin-1 hadrons. For the transverse components, we have,

$$\langle S_T^x \rangle(z, y) = -\frac{8MD(y)}{(2S + 1)zQ} \frac{\sum_q T_3^q(y)G_T(z)}{\sum_q T_0^q(y)D_1(z)}, \quad (27)$$

$$\langle S_T^y \rangle(z, y) = \frac{8MD(y)}{(2S + 1)zQ} \frac{\sum_q T_2^q(y)D_T(z)}{\sum_q T_0^q(y)D_1(z)}, \quad (28)$$

$$\langle S_{LT}^x \rangle(z, y) = -\frac{8MD(y)}{(2S + 1)zQ} \frac{\sum_q T_2^q(y)D_{LT}(z)}{\sum_q T_0^q(y)D_1(z)}, \quad (29)$$

$$\langle S_{LT}^y \rangle(z, y) = \frac{8MD(y)}{(2S + 1)zQ} \frac{\sum_q T_3^q(y)G_{LT}(z)}{\sum_q T_0^q(y)D_1(z)}, \quad (30)$$

where  $D(y) = \sqrt{y(1-y)}$ , and we also define,

$$T_2^q(y) = -c_3^e c_3^q + c_1^e c_1^q B(y), \quad (31)$$

$$T_3^q(y) = c_3^e c_1^q - c_1^e c_3^q B(y). \quad (32)$$

We recall that  $\langle S_T^y \rangle$  is  $P$ -even and naive  $T$ -odd,  $\langle S_T^x \rangle$  is  $P$ -odd and naive  $T$ -even, and  $\langle S_{LT}^y \rangle$  is  $P$ -odd and naive  $T$ -odd. We emphasize that formally vector polarization components such as  $\langle \lambda \rangle$ ,  $\langle S_T^x \rangle$  and  $\langle S_T^y \rangle$  have exactly the same expressions in terms of FFs for spin-1/2 hadrons or vector mesons. This means that Eqs. (25) and (27-28) are the same for hyperons and for vector mesons. They are just given by the corresponding FFs for specified hadrons.

The spin alignment of the vector meson is measured by the 00-component  $\rho_{00}$  of the spin density matrix  $\rho$  in the helicity base. It is directly related to  $\langle S_{LL} \rangle$  by  $\rho_{00} = (1 - 2\langle S_{LL} \rangle)/3$ , which means

$$\rho_{00}(z, y) = \frac{1}{3} - \frac{1}{3} \frac{\sum_q T_0^q(y)D_{1LL}(z)}{\sum_q T_0^q(y)D_1(z)}. \quad (33)$$

We consider the case of integrated over  $\theta$  or  $y$ , and we have

$$\bar{\lambda}(z) = -\frac{2}{2S + 1} \frac{\sum_q c_3^q G_{1L}(z)}{\sum_q c_1^q D_1(z)}, \quad (34)$$

$$\bar{\rho}_{00}(z) = \frac{1}{3} - \frac{1}{3} \frac{\sum_q c_1^q D_{1LL}(z)}{\sum_q c_1^q D_1(z)}, \quad (35)$$

$$\bar{S}_T^x(z) = -\frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_1^q G_T(z)}{\sum_q c_1^e c_1^q D_1(z)}, \quad (36)$$

$$\bar{S}_T^y(z) = -\frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_3^q D_T(z)}{\sum_q c_1^e c_1^q D_1(z)}, \quad (37)$$

$$\bar{S}_{LT}^x(z) = \frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_3^q D_{LT}(z)}{\sum_q c_1^e c_1^q D_1(z)}, \quad (38)$$

$$\bar{S}_{LT}^y(z) = \frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_1^q G_{LT}(z)}{\sum_q c_1^e c_1^q D_1(z)}. \quad (39)$$

We see that at the leading twist, we have only two non-zero components, i.e. the longitudinal polarization  $P_{Lh} = \langle \lambda \rangle$  and  $\rho_{00} = (1 - 2\langle S_{LL} \rangle)/3$ . The transverse polarization exists at twist-3, i.e., it is power suppressed. We also note that there is no twist-3 contribution to  $\langle \lambda \rangle$  or  $\langle S_{LL} \rangle$ . The higher twist corrections to these two components come only from twist-4 or even higher twists [29].

#### D. Hadron polarizations at different energies

At different energies, we need to consider contributions from  $e^+e^- \rightarrow Z \rightarrow VX$ , those from  $e^+e^- \rightarrow \gamma^* \rightarrow VX$  and those from the interference terms. For the longitudinal components, we have

$$\langle \lambda \rangle(y, z) = \frac{2}{2S+1} \frac{\sum_q P_q(y) w_q(y) G_{1L}(z)}{\sum_q w_q(y) D_1(z)}, \quad (40)$$

$$\langle S_{LL} \rangle(y, z) = \frac{3}{2(2S+1)} \frac{\sum_q w_q(y) D_{1LL}(z)}{\sum_q w_q(y) D_1(z)}, \quad (41)$$

and for the transverse components

$$\langle S_T^x \rangle(y, z) = -\frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_x w_q(y) G_T(z)}{\sum_q w_q(y) D_1(z)}, \quad (42)$$

$$\langle S_T^y \rangle(y, z) = \frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_y w_q(y) D_T(z)}{\sum_q w_q(y) D_1(z)}, \quad (43)$$

$$\langle S_{LT}^x \rangle(y, z) = -\frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_y w_q(y) D_{LT}(z)}{\sum_q w_q(y) D_1(z)}, \quad (44)$$

$$\langle S_{LT}^y \rangle(y, z) = \frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_x w_q(y) G_{LT}(z)}{\sum_q w_q(y) D_1(z)}, \quad (45)$$

where  $w_q(y)$  is given by Eq. (15), and  $\Delta_x w_q(y)$  and  $\Delta_y w_q(y)$  are given by

$$\Delta_x w_q(y) = \chi T_3^q(y) + \chi_{int}^q I_3^q(y), \quad (46)$$

$$\Delta_y w_q(y) = \chi T_2^q(y) + \chi_{int}^q I_2^q(y), \quad (47)$$

$$I_2^q(y) = -c_A^e c_A^q + c_V^e c_V^q B(y), \quad (48)$$

$$I_3^q(y) = c_A^e c_V^q - c_V^e c_A^q B(y). \quad (49)$$

After integrating over  $y$ , we obtain

$$\bar{\lambda}(z) = \frac{2}{(2S+1)} \frac{\sum_q \bar{P}_q W_q G_{1L}(z)}{\sum_q W_q D_1(z)}, \quad (50)$$

$$\bar{\rho}_{00}(z) = \frac{1}{3} - \frac{1}{3} \frac{\sum_q W_q D_{1LL}^{q \rightarrow h}(z)}{\sum_q W_q D_1^{q \rightarrow h}(z)}, \quad (51)$$

$$\bar{S}_T^x(z) = -\frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_x W_q G_T(z)}{\sum_q W_q D_1(z)}, \quad (52)$$

$$\bar{S}_T^y(z) = \frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_y W_q D_T(z)}{\sum_q W_q D_1(z)}, \quad (53)$$

$$\bar{S}_{LT}^x(z) = -\frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_y W_q D_{LT}(z)}{\sum_q W_q D_1(z)}, \quad (54)$$

$$\bar{S}_{LT}^y(z) = \frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_x W_q G_{LT}(z)}{\sum_q W_q D_1(z)}, \quad (55)$$

where  $\Delta_x W_q = \pi(\chi c_3^e c_1^q + \chi_{int}^q c_A^e c_V^q)/8$ , and  $\Delta_y W_q = -\pi(\chi c_3^e c_3^q + \chi_{int}^q c_A^e c_A^q)/8$ . We see again that there exist twist-3 transverse polarizations that can be used to study higher twist effects in particular the corresponding higher twist FFs. However, we should also note that at lower energies where electromagnetic interactions dominate, such twist-3 contributions are non-zero only at a given  $y$  but vanish after the integration over  $y$  or  $\theta$  in the entire region. This is consistent with the data available [37]. One can however study such effects by measuring transverse polarizations integrated in a given region of  $\theta$  or  $y$  such as in the forward or backward hemisphere.

### E. QCD evolution equations for $G_{1L}$ and $D_{1LL}$

QCD evolutions for leading twist FFs have been well established and are determined by corresponding DGLAP equations [38–41] with time-like splitting functions [42–44]. We just give the equations that will be used in our numerical estimations in the following. The evolution of the spin transfer  $G_{1L}$  is given by DGLAP in the longitudinally polarized case while that for the

$S_{LL}$ -dependent FF  $D_{1LL}$  is the same as that for unpolarized FF  $D_1$ . They are given by

$$\frac{\partial}{\partial \ln Q^2} G_{1L}^{i \rightarrow h}(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_z^1 \frac{dy}{y} G_{1L}^{j \rightarrow h}\left(\frac{z}{y}, Q^2\right) \Delta P_{ji}(y, \alpha_s), \quad (56)$$

$$\frac{\partial}{\partial \ln Q^2} D_{1LL}^{i \rightarrow h}(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_z^1 \frac{dy}{y} D_{1LL}^{j \rightarrow h}\left(\frac{z}{y}, Q^2\right) P_{ji}(y, \alpha_s), \quad (57)$$

where  $i$  or  $j$  denotes different types of partons such as different flavors of quarks, anti-quarks and gluon. At the leading order in pQCD, the polarized splitting functions are given by [45, 46]

$$\Delta P_{qq}(y) = C_F \left[ \frac{1+y^2}{(1-y)_+} + \frac{3}{2} \delta(1-y) \right], \quad (58)$$

$$\Delta P_{gq}(y) = C_F \frac{1-(1-y)^2}{y}, \quad (59)$$

$$\Delta P_{qg}(y) = [y^2 - (1-y)^2]/2, \quad (60)$$

$$\Delta P_{gg}(y) = N_c \left[ (1+y^4) \left( \frac{1}{y} + \frac{1}{(1-y)_+} \right) - \frac{(1-y)^3}{y} \right] + \frac{11N_c - 2N_f}{6} \delta(1-y), \quad (61)$$

where  $N_c = 3$  and  $C_F = (N_c^2 - 1)/2N_c$ . The unpolarized splitting functions are given by,

$$P_{qq}(y) = \Delta P_{qq}(y), \quad (62)$$

$$P_{gq}(y) = C_F \frac{1+(1-y)^2}{y}, \quad (63)$$

$$P_{qg}(y) = [y^2 + (1-y)^2]/2, \quad (64)$$

$$P_{gg}(y) = N_c \left[ \frac{2y}{(1-y)_+} - 2(y^2 - y - \frac{1}{y} + 1) \right] + \frac{11N_c - 2N_f}{6} \delta(1-y). \quad (65)$$

The purpose of our studies in this paper is not to make a global fit for the polarized FFs but only a demonstration of two distinctly different behaviors in energy dependences of hadron polarizations in  $e^+e^-$ -annihilation. We therefore limit ourselves to the  $e^+e^-$  data and to the next-to-leading order in pQCD where only leading order splitting functions given above are used.

### III. NUMERICAL RESULTS FOR $P_{L\Lambda}$ AND $\bar{\rho}_{00}^{K^*}$

In this section, we use  $P_{L\Lambda}$  and  $\bar{\rho}_{00}^{K^*}$  as representative examples and present the numerical results on the energy dependences. We choose them as examples since we have data for them from LEP [1–5].

### A. Parameterizations and QCD evolutions of $G_{1L}$ and $D_{1LL}$

Because of decay contributions, polarization of  $\Lambda$  hyperon is much more involved than other hyperons and/or vector mesons. In general, the leading twist FF for a quark to a baryon,  $q \rightarrow B_i$ , can be written as the sum of a direct fragmentation and a decay contribution part, i.e.,

$$D_1^{q \rightarrow B_i}(z) = D_{1,dir}^{q \rightarrow B_i}(z) + \sum_{j \neq i} R_D^{ji} \int dz' K_D^{ji}(z, z') D_1^{q \rightarrow B_j}(z'), \quad (66)$$

$$G_{1L}^{q \rightarrow B_i}(z) = G_{1L,dir}^{q \rightarrow B_i}(z) + \sum_{j \neq i} R_D^{ji} \int dz' K_D^{ji}(z, z') t_D^{ji}(z, z') G_{1L}^{q \rightarrow B_j}(z'), \quad (67)$$

where  $R_D^{ji}$  is the decay branch ratio of  $B_j \rightarrow B_i + X$ ,  $K_D^{ji}(z, z')$  is the probability for a  $B_j$  with  $z'$  to decay into a  $B_i$  with  $z$ , and  $t_D^{ji}(z, z')$  is the spin transfer factor in the decay process.

Numerical results show that, for the  $\Lambda$  hyperon, the contributions from  $\Sigma^0$  and  $\Xi^{0,-}$  are sizable [7, 8]. However, since there is no suitable data for  $\Sigma^0$  or  $\Xi$  polarization in  $e^+e^-$  available yet, it is impossible to make such a detailed analysis. On the other hand, the energy dependences of the hadron polarizations that we will study in this paper come mainly from the QCD evolution of FFs and the energy dependence of the polarization of the quark produced at the  $e^+e^-$ -annihilation vertex. We would expect that the influences from the decay contributions on the energy dependence are not very large. Hence, in this paper, as a rough estimation, we simply parameterize the final  $D_1^{q \rightarrow \Lambda}(z)$  and  $G_{1L}^{q \rightarrow \Lambda}(z)$ , and evolve them to other energies using DGLAP equations given by Eq. (56).

For vector mesons such as  $K^*$ , the decay contributions are negligible. So we need to simply parameterize and evolve the corresponding  $D_1$  and  $D_{1LL}$  to obtain  $\bar{S}_{LL}$  at different energies.

A number of parameterizations exist in literature for the FF  $D_1(z)$  for unpolarized hadron production such as pion, Kaon, proton and  $\Lambda$  [47]. We just take the most recent parameterizations AKK08 given in [48] for  $\Lambda$ . For polarized FFs, we make a simple parametrization in a similar scheme as DSV [49], i.e., for the  $s$ -quark fragmentation, we take

$$G_{1L}^{s \rightarrow \Lambda}(z) = z^a D_1^{s \rightarrow \Lambda}(z), \quad (68)$$

while for  $u$  and  $d$ -quark, we take

$$G_{1L}^{q \rightarrow \Lambda}(z) = N z^a D_1^{q \rightarrow \Lambda}(z), \quad (69)$$

where  $q = u$  or  $d$ , and limit the parameters  $a > 0$  and  $|N| \leq 1$ . By fitting data from LEP for  $\Lambda$  polarization [1, 2], we fix the parameters as  $a = 0.932$  and  $N = 0.136$ . The result of the fit for  $\Lambda$

polarization is shown by the solid line in Fig. 2. The obtained  $G_{1L}^{q \rightarrow \Lambda}(z, Q^2)$  at  $Q = M_Z$  for  $q = u, d$  and  $s$  are shown in Fig. 3. We see that in general  $G_{1L}^{s \rightarrow \Lambda}(z, Q^2)$  is positive and much larger than  $G_{1L}^{u \rightarrow \Lambda}(z, Q^2)$  or  $G_{1L}^{d \rightarrow \Lambda}(z, Q^2)$ . The small difference between  $G_{1L}^{u \rightarrow \Lambda}(z, Q^2)$  and  $G_{1L}^{d \rightarrow \Lambda}(z, Q^2)$  comes from that between  $D_1^{u \rightarrow \Lambda}(z, Q^2)$  and  $D_1^{d \rightarrow \Lambda}(z, Q^2)$  from AKK08 [48].

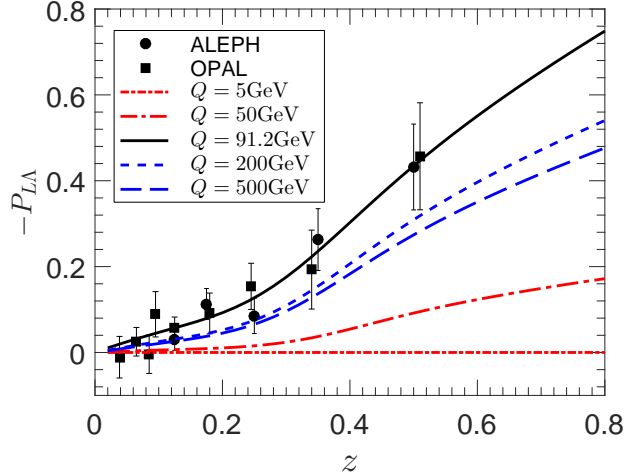


FIG. 2: (color online) Longitudinal polarization of  $\Lambda$  in  $e^+e^- \rightarrow \Lambda X$  at high energies. The LEP data are taken from [1, 2]. The solid line is the fit described in the text while those at other energies are calculated results using DGLAP for FFs and energy dependence of  $P_q$ .

For the vector meson  $K^*$ , there is no parameterization of FF in the unpolarized case available yet. We therefore make a simple parameterization by using those for  $K^\pm$  from AKK08 [48] and by parameterizing the data for the ratio of  $K^*$  to  $K$  as given in [50]. The  $z$ -dependence for the ratio is taken as linear, i.e.,  $D_1^{K^*}(z)/3D_1^{K^+}(z) = 0.2z + 0.1$ , and is assumed to be the same for different flavors.

For the  $S_{LL}$ -dependent FFs,  $D_{1LL}^{q \rightarrow K^*}(z)$ , inspired by the almost linear  $z$ -dependence of data of  $\rho_{00}$  [3], we parameterize them as

$$D_{1LL}^{q \rightarrow K^*}(z) = c_1 D_1^{q \rightarrow K^*}(z), \quad (70)$$

for un-favored fragmentations and

$$D_{1LL}^{q \rightarrow K^*}(z) = (c_1 + c_2 z) D_1^{q \rightarrow K^*}(z), \quad (71)$$

for favored fragmentations. By fitting the available  $z$ -dependence data on the spin alignment of  $K^*$  from OPAL [3], we fix the parameters as  $c_1 = 0.186$  and  $c_2 = -1.35$ . The fitted curve is presented

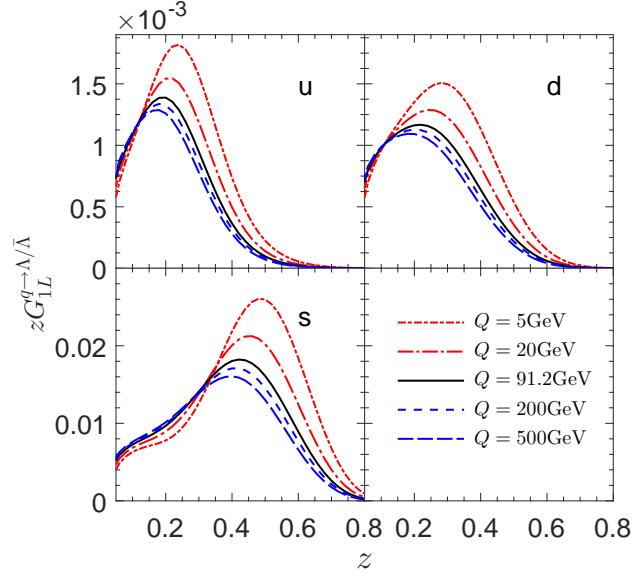


FIG. 3: (color online) The longitudinal spin transfer fragmentation function  $G_{1L}(z, Q^2)$  for  $q \rightarrow \Lambda$  as a function of  $z$  for different flavors of  $q$  at different values of  $Q$ . The solid lines are obtained by fitting data for  $P_{L\Lambda}$  at  $Q = M_Z$ , and the others are obtained using DGLAP with leading order splitting functions described in the text.

in Fig. 4. The obtained  $D_{1LL}^{q \rightarrow K^*0}(z, Q^2)$  at  $Q = M_Z$  for  $q = u, d$  or  $s$  is given by the solid line in Fig. 5.

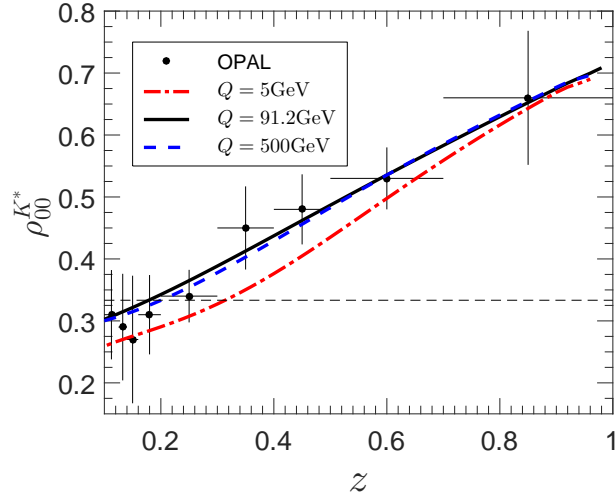


FIG. 4: (color online) Spin alignment of  $K^*$  as a function of  $z$ . The solid line is the fit described in the text while those at other energies are calculated results using DGLAP for FFs. The data points are from OPAL at LEP and are taken from [3].

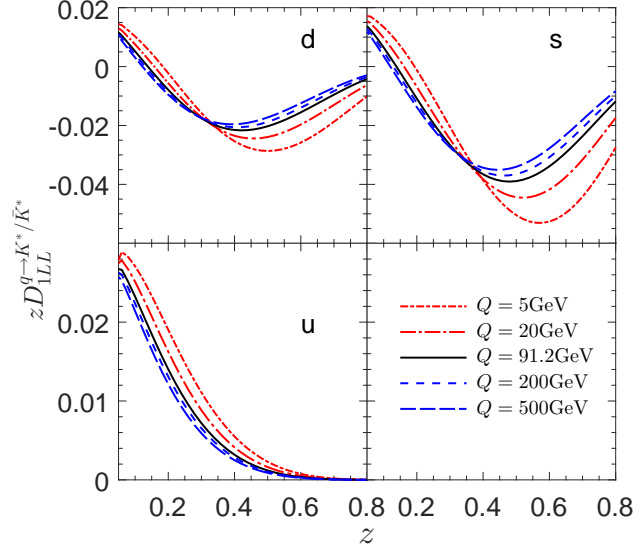


FIG. 5: (color online) The  $S_{LL}$  dependent fragmentation function  $D_{1LL}(z, Q^2)$  for  $q \rightarrow K^*$  as a function of  $z$  for different flavors of  $q$  at different values of  $Q$ . The solid lines are obtained by fitting data for  $\rho_{00}^{K^*}$  at  $Q = M_Z$ , the others are obtained using DGLAP with leading order splitting functions described in the text.

Because the data (see Fig. 4) for  $\rho_{00}^{K^*}$  are larger than  $1/3$  in the large  $z$  region, the  $S_{LL}$ -dependent FF  $D_{1LL}(z, Q^2)$  should be negative in the corresponding  $z$  region. At small  $z$ ,  $\rho_{00}$  is smaller than  $1/3$ , which implies a positive  $D_{1LL}(z, Q^2)$ . These features are shown clearly in Fig. 5, where we can see that for favored fragmentations,  $D_{1LL}(z, Q^2)$  are negative at larger  $z$  while those for unfavored fragmentations also play the important role in the small  $z$  region and are positive.

Having obtained these parameterizations of FFs at  $Q = M_Z$ , we input them into the corresponding DGLAP evolution equations Eqs. (56) and (57) and evolve to other energies. By solving these equations numerically, we obtain the results for  $G_{1L}^{q \rightarrow \Lambda}(z, Q^2)$  and  $D_{1LL}^{q \rightarrow K^*}(z, Q^2)$  at different values of  $Q$ . In Fig. 3, we show  $G_{1L}^{q \rightarrow \Lambda}(z, Q^2)$  as a function of  $z$  at different values of  $Q$ . In Fig. 6, we show  $G_{1L}^{q \rightarrow \Lambda}(z, Q^2)$  as a function of  $Q$  at different values of  $z$ . Similarly, we show  $D_{1LL}^{q \rightarrow K^*}(z, Q^2)$  as a function of  $z$  at different values of  $Q$  in Fig. 5 and as a function of  $Q$  at different values of  $z$  in Fig. 7. Here, in obtaining the results given in the figure, we neglect the contribution from gluon fragmentation to spin dependent FFs, i.e., we take  $G_{1L}^{g \rightarrow \Lambda}(z, Q^2) = D_{1LL}^{g \rightarrow K^*}(z, Q^2) = 0$ . In Figs. 6 and 7, to show  $Q$ -dependences more clearly, we scale the FFs by those at  $Q_0^2 = 10 \text{ GeV}^2$ .

From Fig. 3, we see that the peaks for  $zG_{1L}(z, Q^2)$  shift towards smaller  $z$  for larger  $Q$ . In the large  $z$  region,  $G_{1L}(z, Q^2)$  decreases with increasing  $Q$  while for small  $z$  it increases with increasing  $Q$ . This former is shown more obviously in Fig. 6. In the large  $z$  region, we see the same tendency



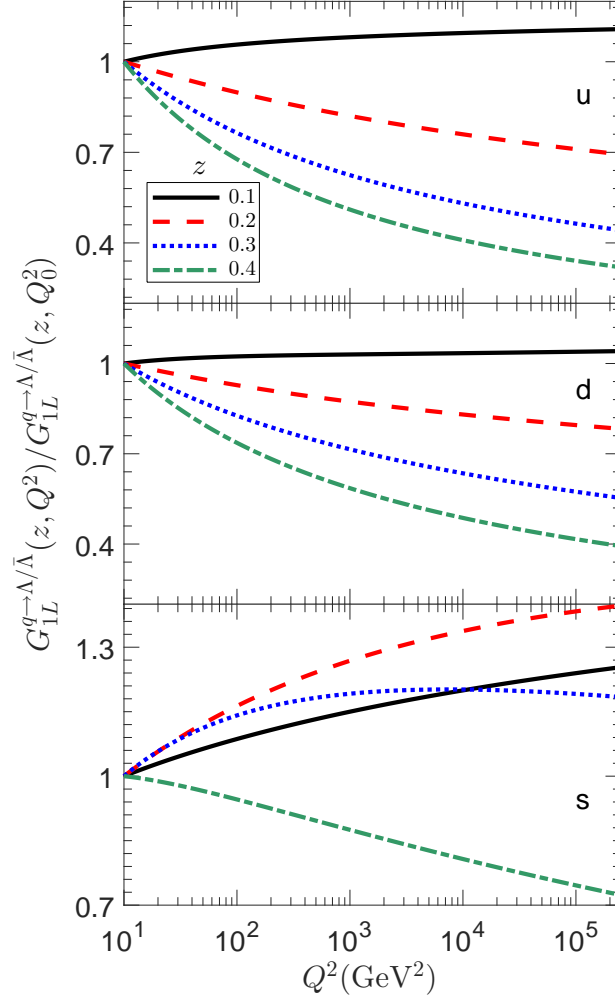


FIG. 6: (color online) QCD evolved  $G_{1L}(z, Q^2)$  for  $q \rightarrow \Lambda$  as function of  $Q$  at different  $z$  divided by the corresponding value at  $Q_0^2 = 10\text{GeV}^2$ .

for  $D_{1LL}(z, Q^2)$  as a function of  $Q$  from Fig. 5 and Fig. 7, i.e. the magnitude of  $D_{1LL}(z, Q^2)$  also decreases with increasing  $Q$  for large  $z$  values. However, because of the crossover with zero at  $z \sim 0.1 - 0.2$ , we see relatively rapid changes for the corresponding  $D_{1LL}(z, Q^2)$ .

However, as expected, the magnitudes of these FFs do not change with  $Q$  as drastically as  $P_q$  does (see Fig 1). We therefore expect that the  $Q$ -dependence of  $P_{L\Lambda}$  should be dominated by that of  $P_q$ .

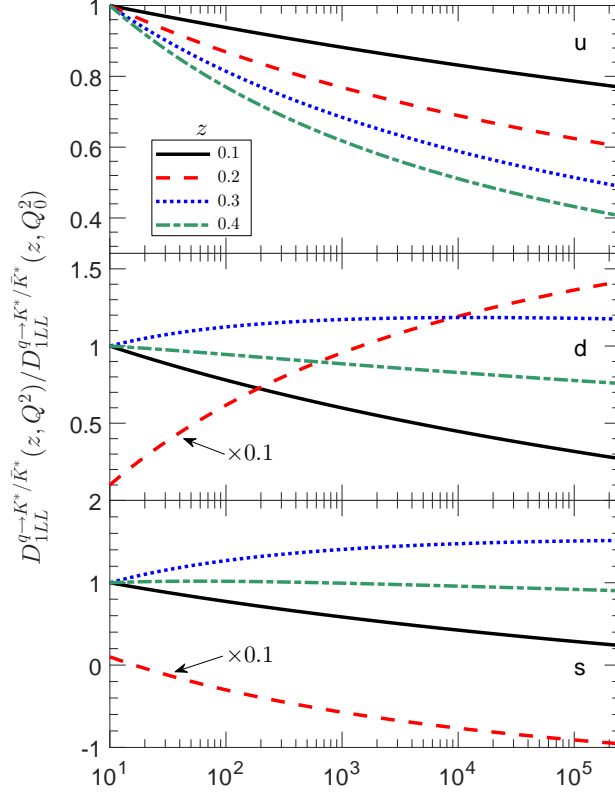


FIG. 7: QCD evolved  $D_{1LL}(z, Q^2)$  for  $q \rightarrow K^*$  as function of  $Q$  at different  $z$  divided by the corresponding value at  $Q_0^2 = 10\text{GeV}^2$ .

### B. Energy dependence of $P_\Lambda$ and $\bar{\rho}_{00}^{K^*}$

By inserting these results for FFs at different  $Q$ , we obtained  $P_{L\Lambda}$  and  $\bar{\rho}_{00}^{K^*}$  at different energies  $\sqrt{s} = Q$ . We plot the results in Figs. 2 and 4 respectively.

From Figs. 2 and 4, we see clearly that there is a strong energy dependence for  $P_{L\Lambda}$ , whereas that for  $\bar{\rho}_{00}^{K^*}$  is quite weak. The former comes mainly from the energy dependence of  $P_q$  while the latter comes mainly from QCD evolution of FFs. To show this more explicitly, we plot  $P_{L\Lambda}$  at a given  $z$  as a function of  $Q$  in the same figure as  $\bar{P}_q$  in Fig. 8.

From Fig. 8, we explicitly see that  $P_{L\Lambda}$  behaves in very much the same way as  $P_q$  as functions of  $Q$ . This shows clearly that the energy dependence of  $P_{L\Lambda}$  is dominated by that of  $P_q$ . We see in particular that, just as  $\bar{P}_q$ ,  $\bar{P}_{L\Lambda}$  changes very fast with energy and goes to zero when  $Q$  deviates from  $Q = M_Z$  for  $Q < M_Z$ . This is because at smaller  $Q$ , electromagnetic interaction becomes dominant and weak interaction via exchange of  $Z$ -boson becomes negligible rapidly. Whereas at large  $Q$ ,

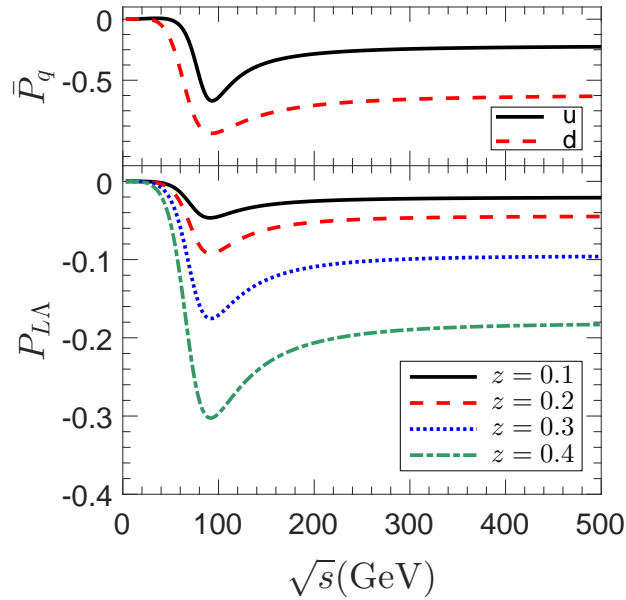


FIG. 8: (color online) Energy dependence of the longitudinal  $\Lambda$  polarization in  $e^+e^-$  annihilation.

although smaller than that at the  $Z$ -pole, it is still sizable and becomes quite flat with increasing  $Q$ . The results show in particular that at BES or BELLE energies,  $P_{L\Lambda}$  should be negligibly small. Furthermore, from the results presented in Sec. II D such as Eqs. (40-45), we see that there is no twist-3 contribution to  $P_{L\Lambda}$  but there can be twist-3 contribution to the transverse components. Higher twist contributions to  $P_{L\Lambda}$  come only from twist-4 or even higher twist [29]. This implies that at BES energies, the transverse components could even become larger than the longitudinal component for a given region of  $\theta$  or  $y$ .

In contrast to  $P_{L\Lambda}$ ,  $\rho_{00}$  changes with  $Q$  quite weakly and remains sizable even at BES energies. This is a clear prediction that can be tested by future experiments.

#### IV. SUMMARY AND OUTLOOK

Using the longitudinal polarization  $P_{L\Lambda}$  of  $\Lambda$  hyperon and the spin alignment  $\rho_{00}^{K^*}$  of  $K^{*0}$  as representative examples, we demonstrate the two very different behaviors in energy dependences of hadron polarizations in  $e^+e^-$  annihilations. The results show clearly that  $P_{L\Lambda}$  has a very strong energy dependence due to its direct dependence on the initial longitudinal polarization  $P_q$  of the quark  $q$ , while  $\rho_{00}^{K^*}$  has a rather weak energy dependence since it is independent of  $P_q$ . The former is dominated by the energy dependence of  $P_q$  while the latter comes mainly from the QCD evolutions

of the FFs. We have presented the results at the leading twist with pQCD evolution at the leading order. In view that the measurements of both  $P_{L\Lambda}$  and  $\rho_{00}^{K^*}$  can in principle be easily carried out in experiments at BES or BELLE, we think that this provides a good place to test QCD evolutions of FFs and/or to check whether higher twist effects are important.

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